Syntax-driven analysis of context-free languages with respect to fuzzy relational semantics

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Motivation

Lotfi Zadeh envisions a precisiated natural language (PNL) as a crucial component of a computational theory of perceptions (CTP).

Such a PNL would have to assign to a statement like ***Carol lives in a small city near San Francisco*** a fuzzy set to represent its meaning.

Motivation

Carol lives in a small city near San Francisco. (example due to Zadeh)

To what extent have we succeeded in systematically determining the meaning of such statements?

What linguistic issues need to be addressed if PNL is to become a reality?

Ordering-based semantics

What exactly is it that a fuzzy set represents in a theory of natural language semantics?

A first approach: The meaning of a vague expression **is** a **fuzzy set**.

Alternatively: meaning is the **ordering imposed** on the domain **by a fuzzy set**.

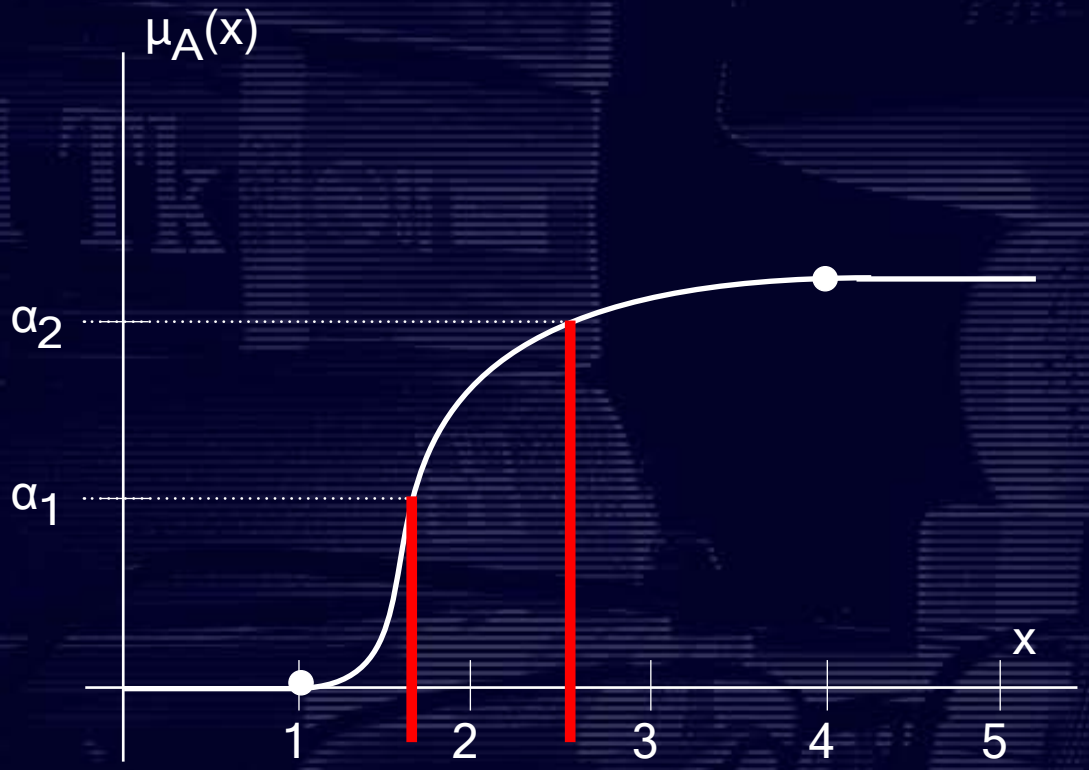
Ordering-based semantics

Problem: no universal intuitions about sets.

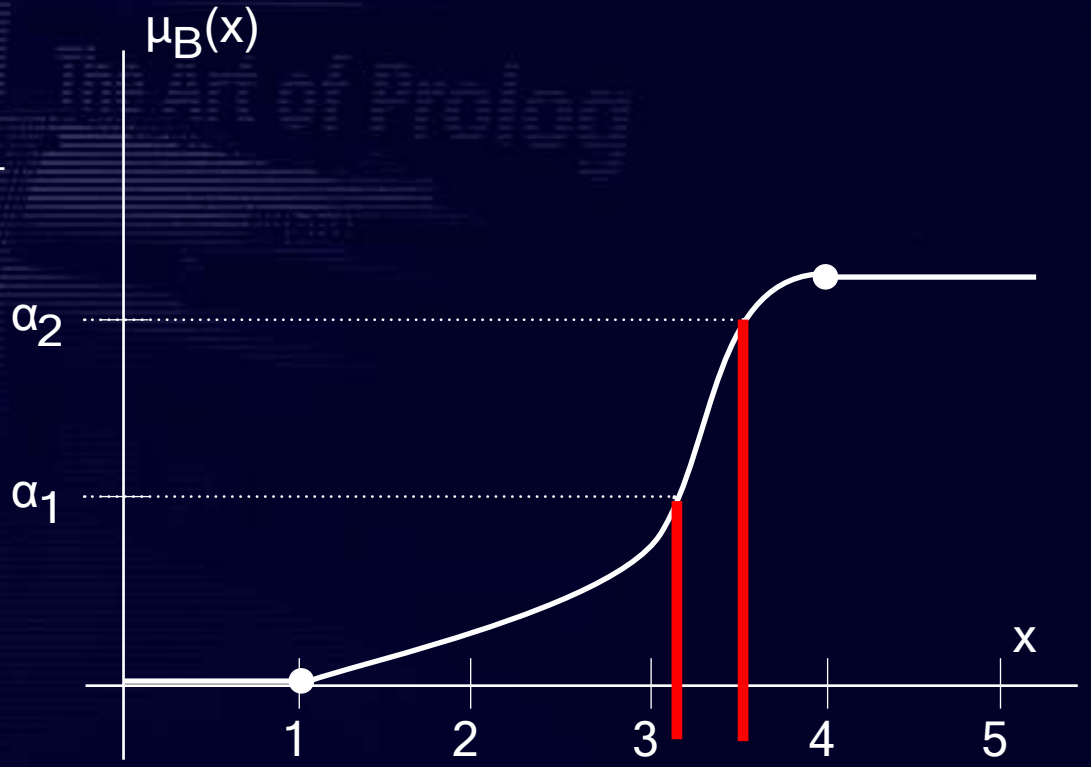
The decision boundary for a ***tiny*** city is sometimes placed higher than for a ***small*** one, for different subjects (Bergmair 2006)

very only shifts decision boundaries when subjects can directly contrast them. (Cliff 1988, Smith et al. 1988, O‘ Muircheartaigh et al. 1993, Wright et al. 1995)

Ordering-based semantics



An ordering may be **robust** to shifting decision boundaries.



$A \neq B$,

$\mu_A(x_1) \geq \mu_A(x_2)$
iff $\mu_B(x_1) \geq \mu_B(x_2)$

Compositionality

How can we establish a systematic relation between an infinitude of syntactic surface forms and semantic representations?

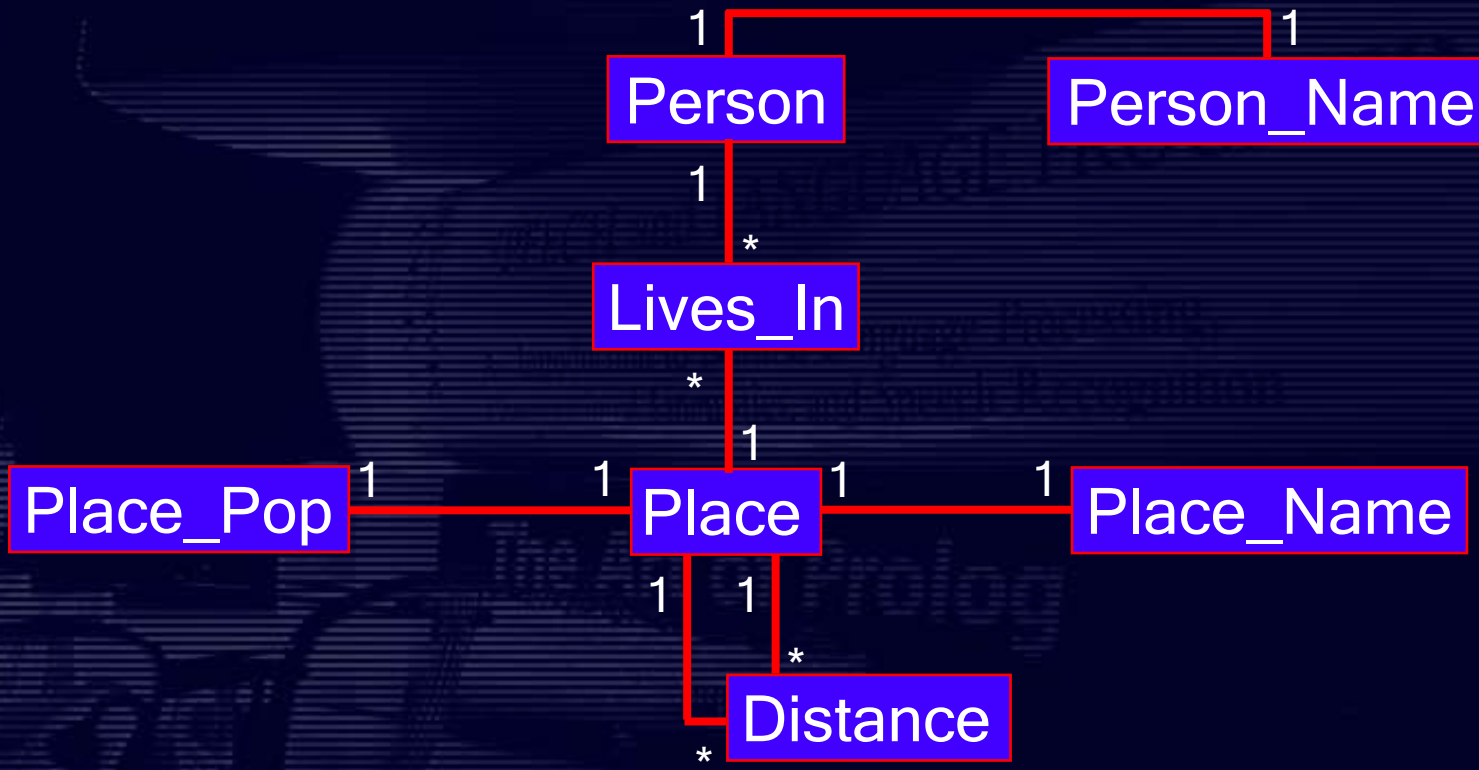
dominant approach: **compositionality**.

In a grammar, each syntactic production corresponds to exactly one semantic production.

Compositionality

(1)	S → NP VP	$\mathsf{S}(x) := \sup_y\{T(\mathsf{NP}(y), \mathsf{VP}(x,y))\}$
(2)	VP → V PP	$\mathsf{VP}(x,\lambda_y) := \sup_z\{T(\mathsf{V}(x,\lambda_y,z), \mathsf{PP}(z))\}$
(3)	VP → V NP	$\mathsf{VP}(x,\lambda_y) := \sup_z\{T(\mathsf{V}(x,\lambda_y,z), \mathsf{NP}(z))\}$
(4)	NP → Nom	$\mathsf{NP}(x) := \mathsf{Nom}(x)$
(5)	NP → Det N'	$\mathsf{NP}(x) := \mathsf{N'}(x)$
(6)	N' → N	$\mathsf{N'}(x) := \mathsf{N}(x)$
(7)	N' → AP N	$\mathsf{N'}(x) := T(\mathsf{AP}(x), \mathsf{N}(x))$
(8)	N' → N' PP	$\mathsf{N'}(x) := T(\mathsf{N'}(x), \mathsf{PP}(x))$
(9)	AP → Adj	$\mathsf{AP}(x) := \mathsf{Adj}(x)$
(10)	AP → very AP	$\mathsf{AP}(x) := (\mathsf{AP}(x))^2$
(11)	PP → in NP	$\mathsf{PP}(x) := \mathsf{NP}(x)$
(12)	PP → near NP	$\mathsf{PP}(x) := \sup_y\{T(\mathsf{NP}(y), \max(\min(\frac{50km-d}{50km-20km}, 1), 0)) \mid \mathsf{Place\_Distance}(x,y,d)\}$
(13)	V → lives	$\mathsf{V}(x,\lambda_y,\lambda_z) := 1.0$ if $\mathsf{Lives\_In}(x,\lambda_y,\lambda_z)$, 0.0 otherwise
(14)	Adj → small	$\mathsf{Adj}(x) := \max(\min(\frac{20000-p}{20000-10000}, 1), 0) \mid \mathsf{Place\_Population}(x,p)$
(15)	N → city	$\mathsf{N}(x) := 1.0$ if $\mathsf{Place}(x)$, 0.0 otherwise
(16)	Nom → Carol	$\mathsf{Nom}(x) := 1.0$ if $\mathsf{Person\_Name}(x, \mathsf{carol})$, 0.0 otherwise
(17)	Nom → Frank	$\mathsf{Nom}(x) := 1.0$ if $\mathsf{Person\_Name}(x, \mathsf{frank})$, 0.0 otherwise
(18)	Nom → San Fr.	$\mathsf{Nom}(x) := 1.0$ if $\mathsf{Place\_Name}(x, \mathsf{san\ francisco})$, 0.0 otherwise
(19)	Det → a	$\mathsf{Det}(x) := 0$

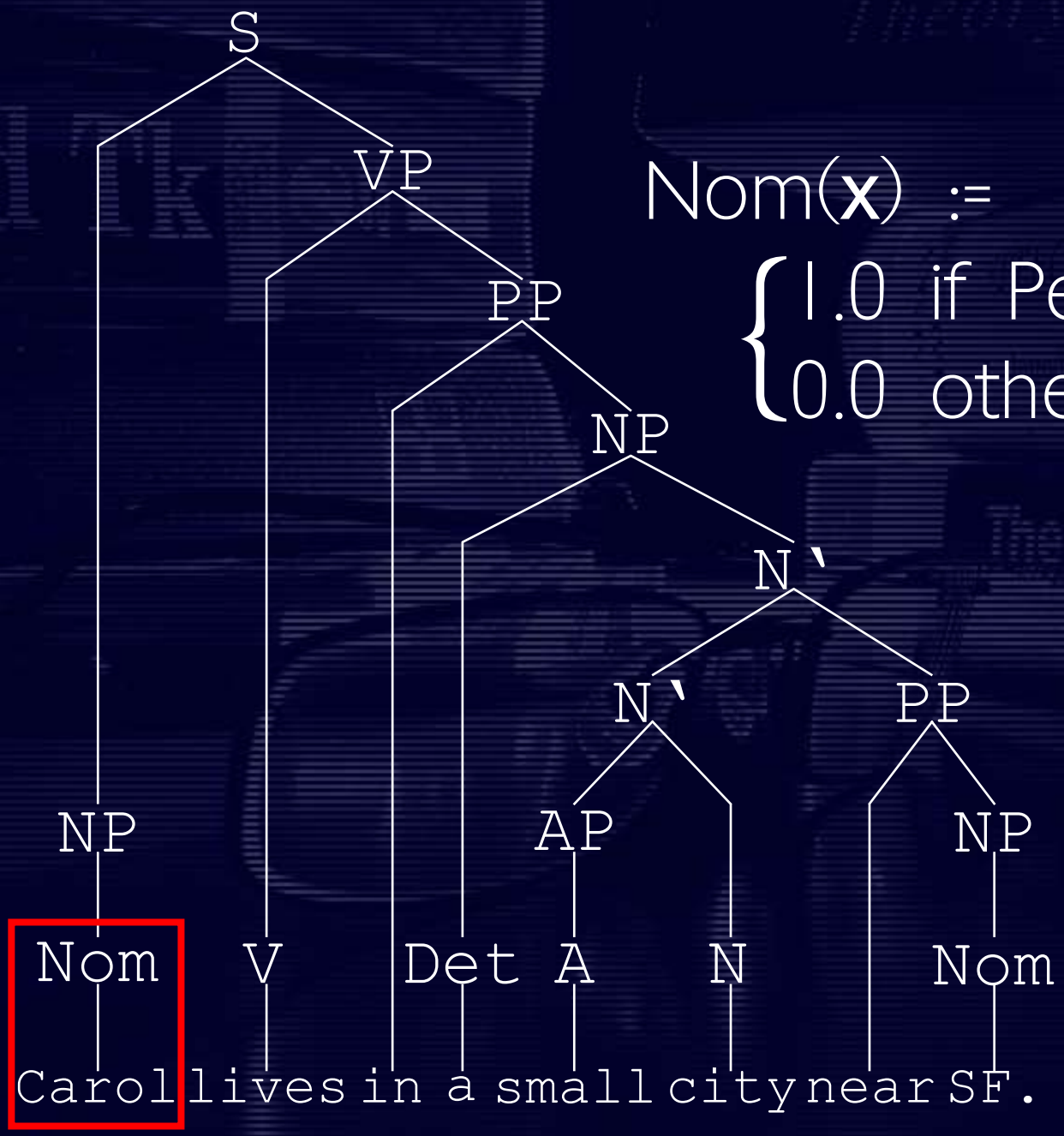
Compositionality



Problem:

What is the meaning of ***Carol lives in a small city near San Francisco*** w.r.t. this database?

Compositionality

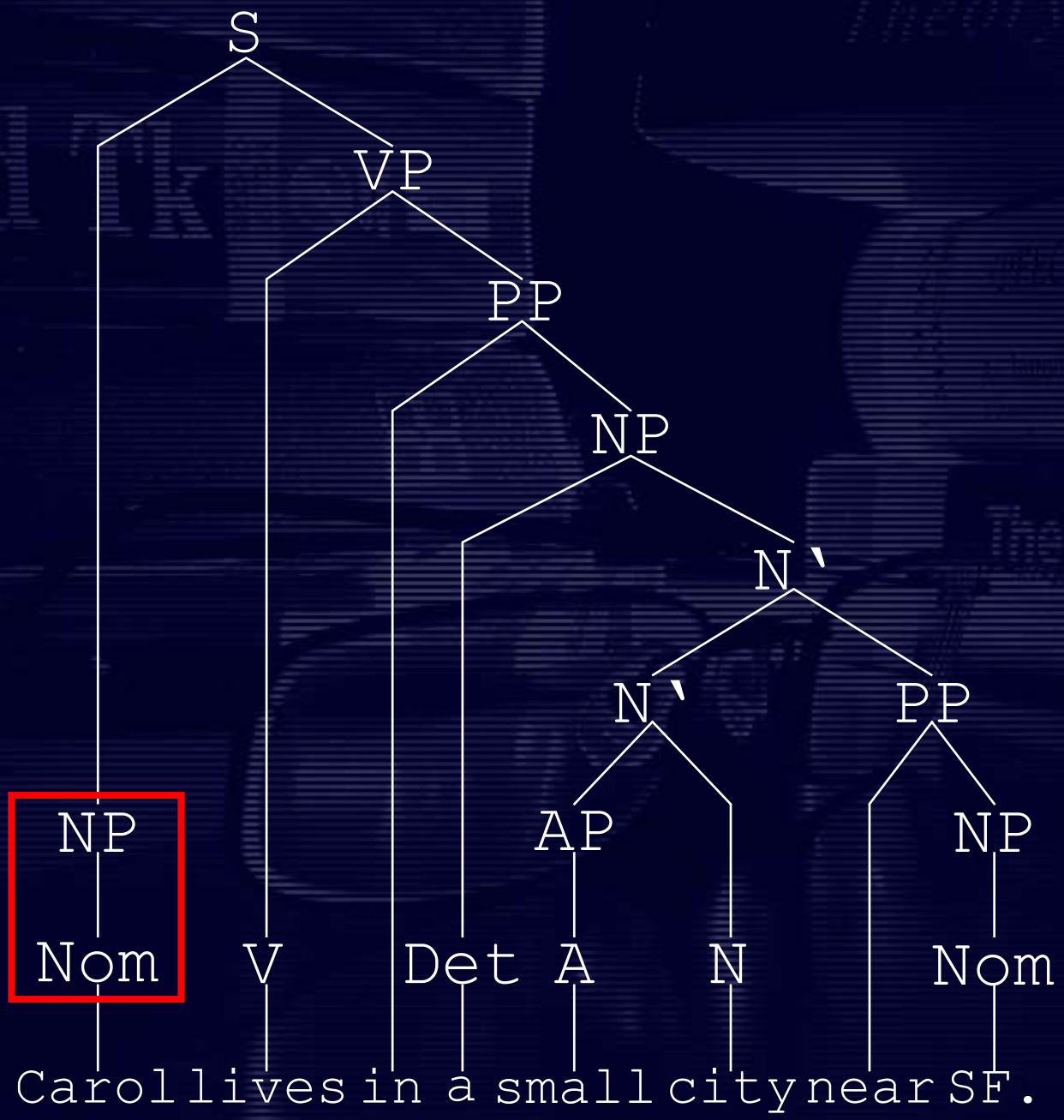


Nom -> Carol

Nom(**x**) :=

{ 1.0 if Person_Name(**x**,Carol),
  0.0 otherwise.

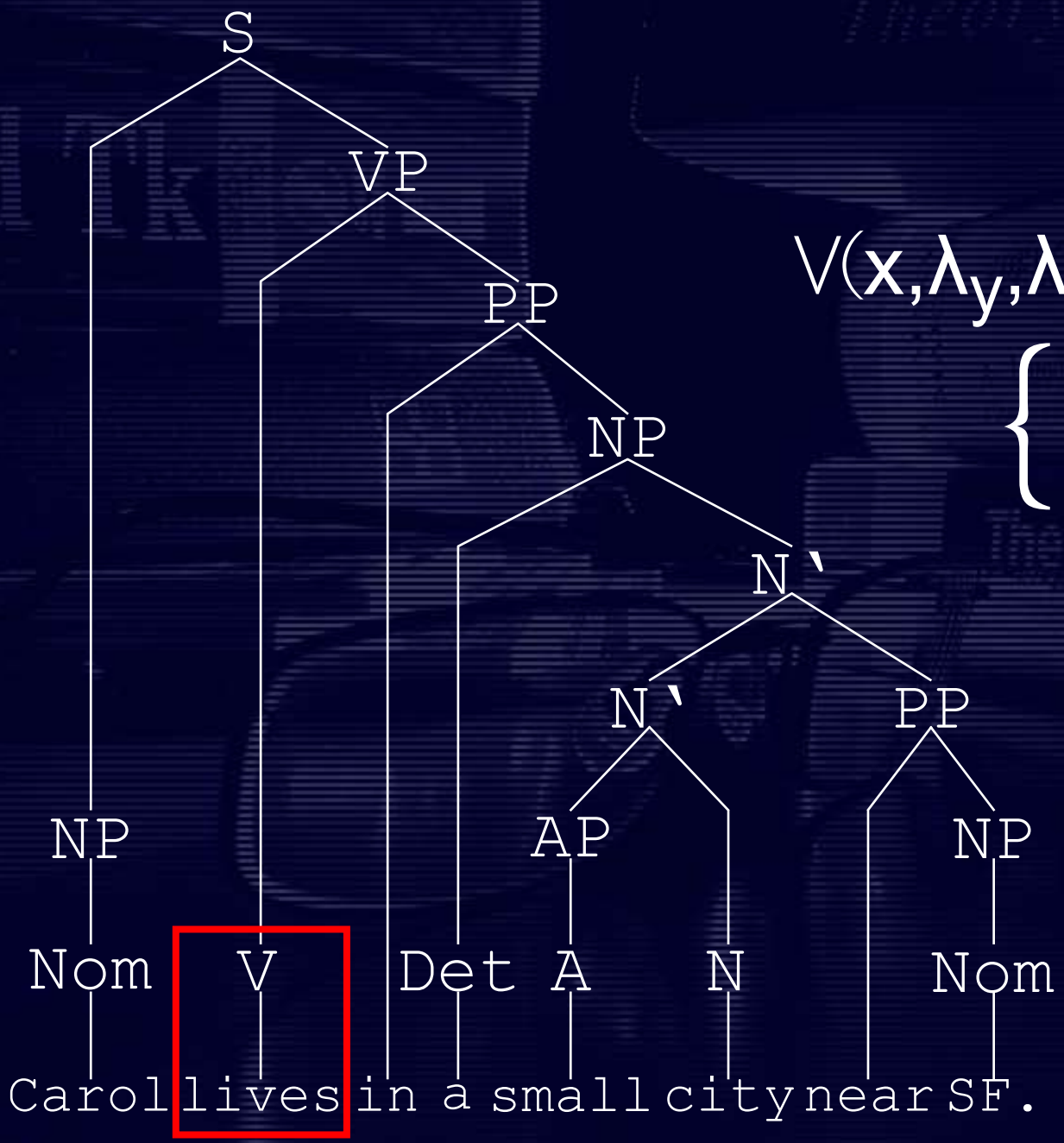
Compositionality



NP -> Nom

NP(**x**) := Nom(**x**)

Compositionality

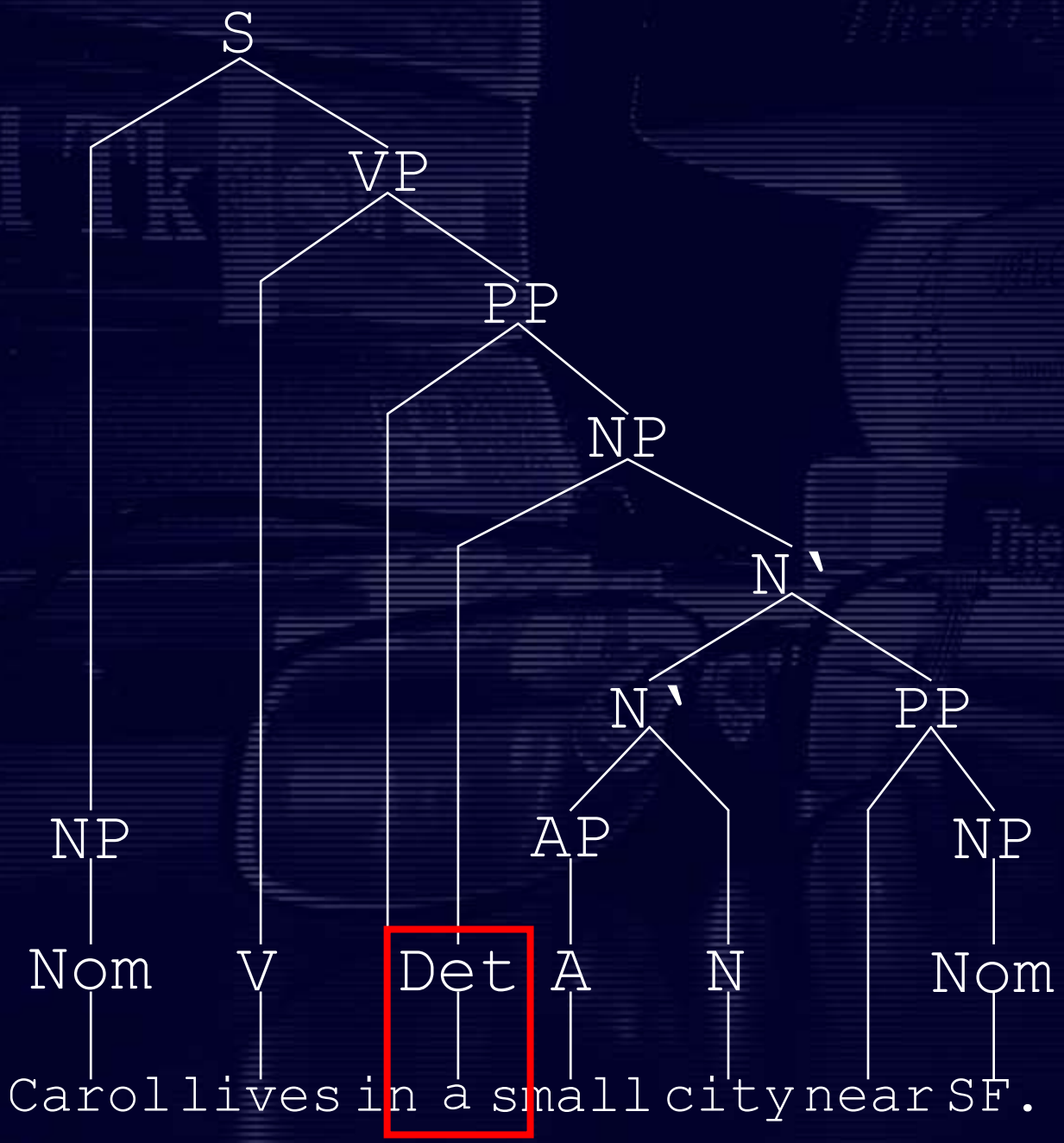


V -> lives

$V(\mathbf{x},\lambda_y,\lambda_z)$:=

$$\begin{cases} 1.0 & \text{if } \text{Lives\_In}(\mathbf{x},\lambda_y,\lambda_z), \\ 0.0 & \text{otherwise.} \end{cases}$$

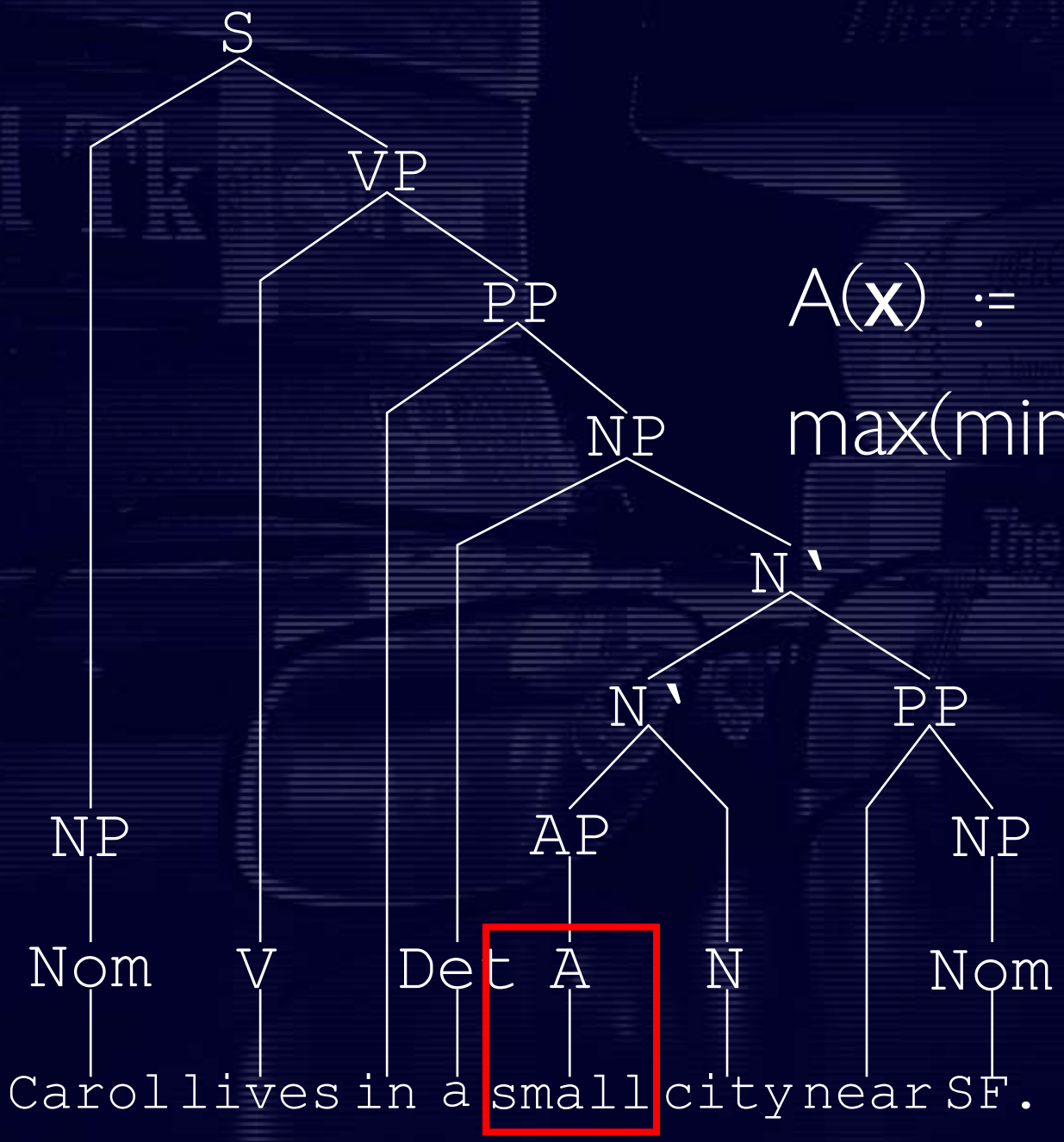
Compositionality



Det -> a

Det(**x**) := 0

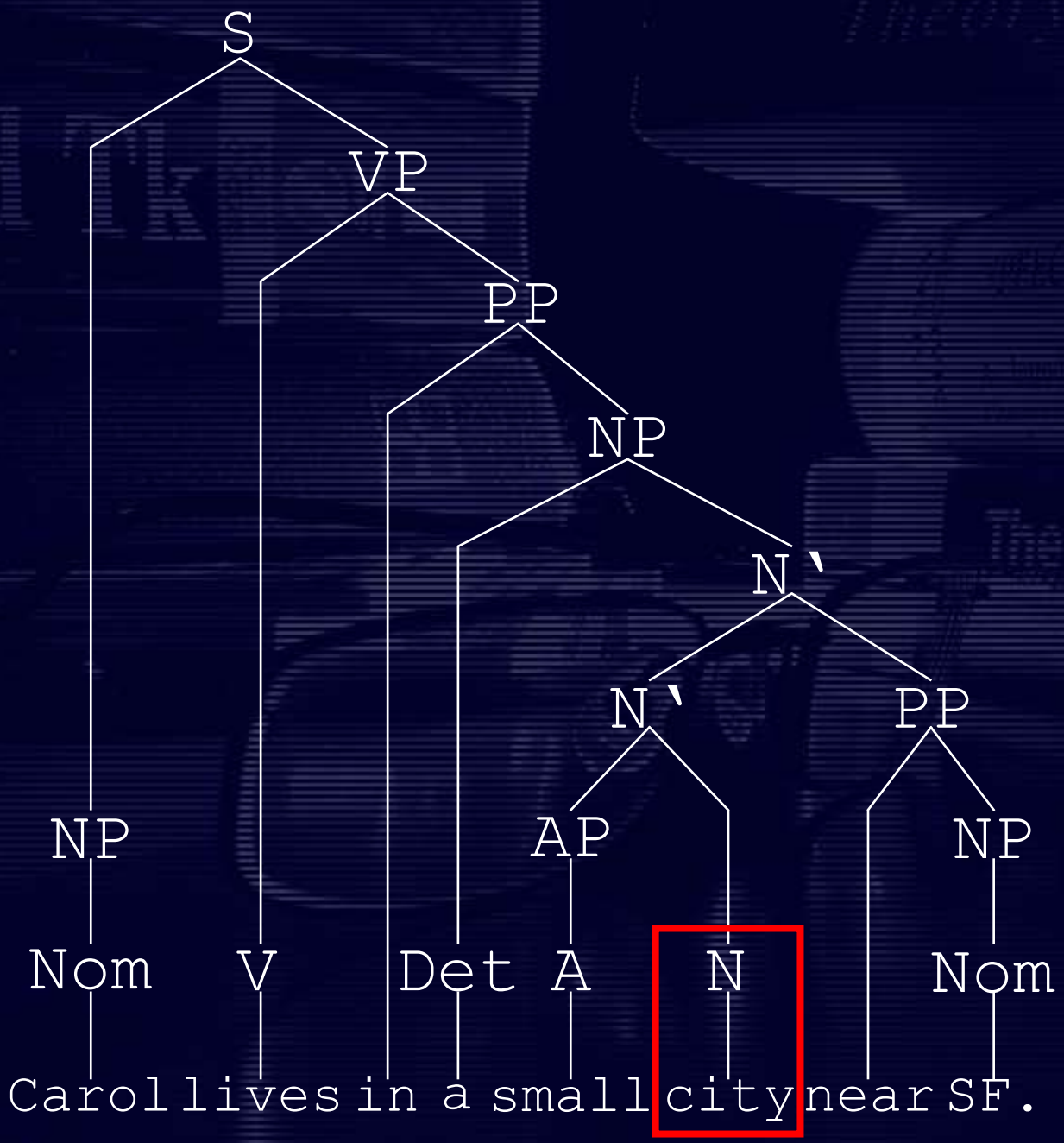
Compositionality



A -> small

$$A(\mathbf{x}) := \max(\min(\frac{20000 - p}{20000 - 10000}, 1), 0) \mid \text{Place\_Population}(\mathbf{x}, \mathbf{p})$$

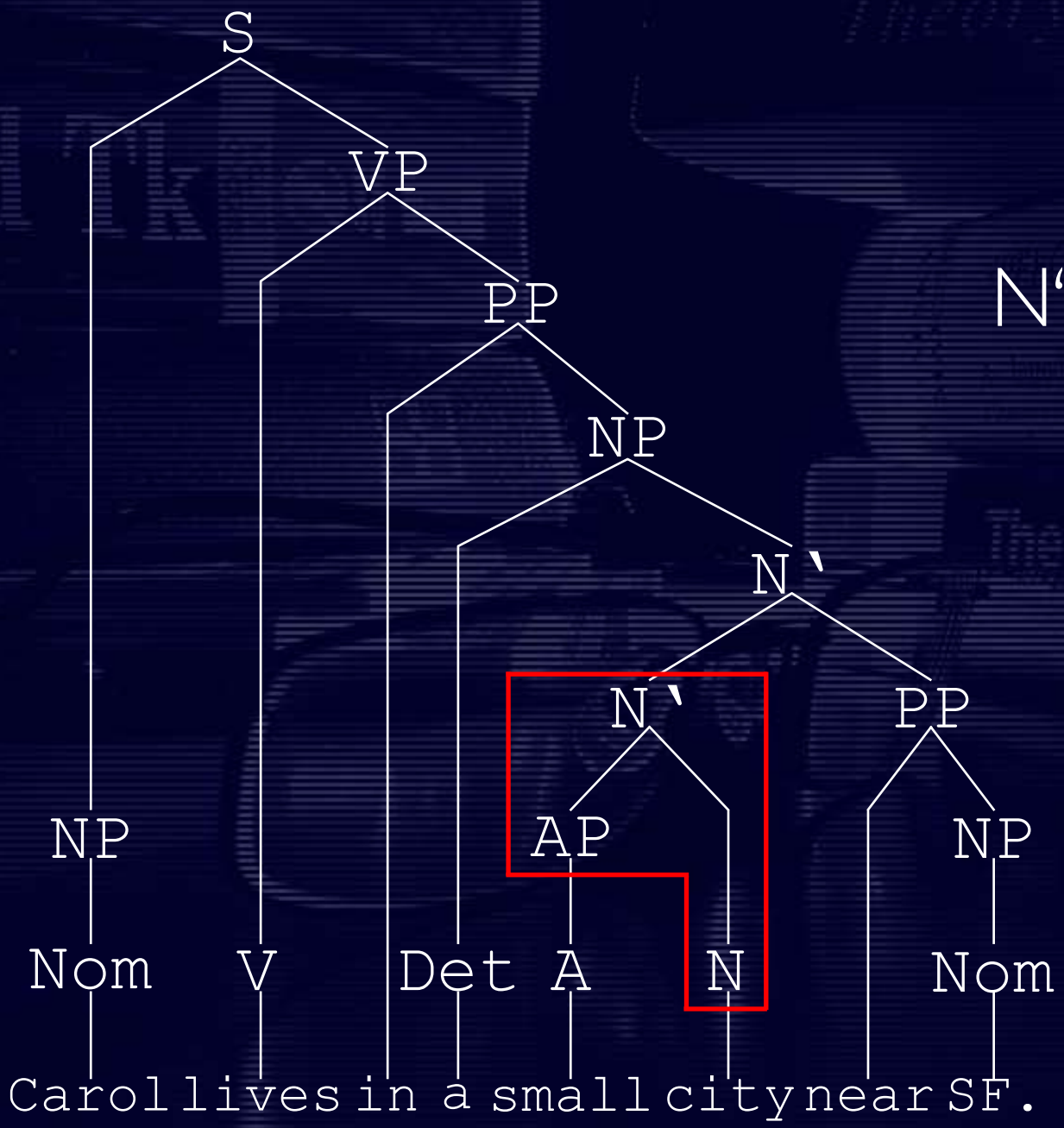
Compositionality



N -> city

$$N(\mathbf{x}) := \begin{cases} 1.0 & \text{if Place}(\mathbf{x}), \\ 0.0 & \text{otherwise.} \end{cases}$$

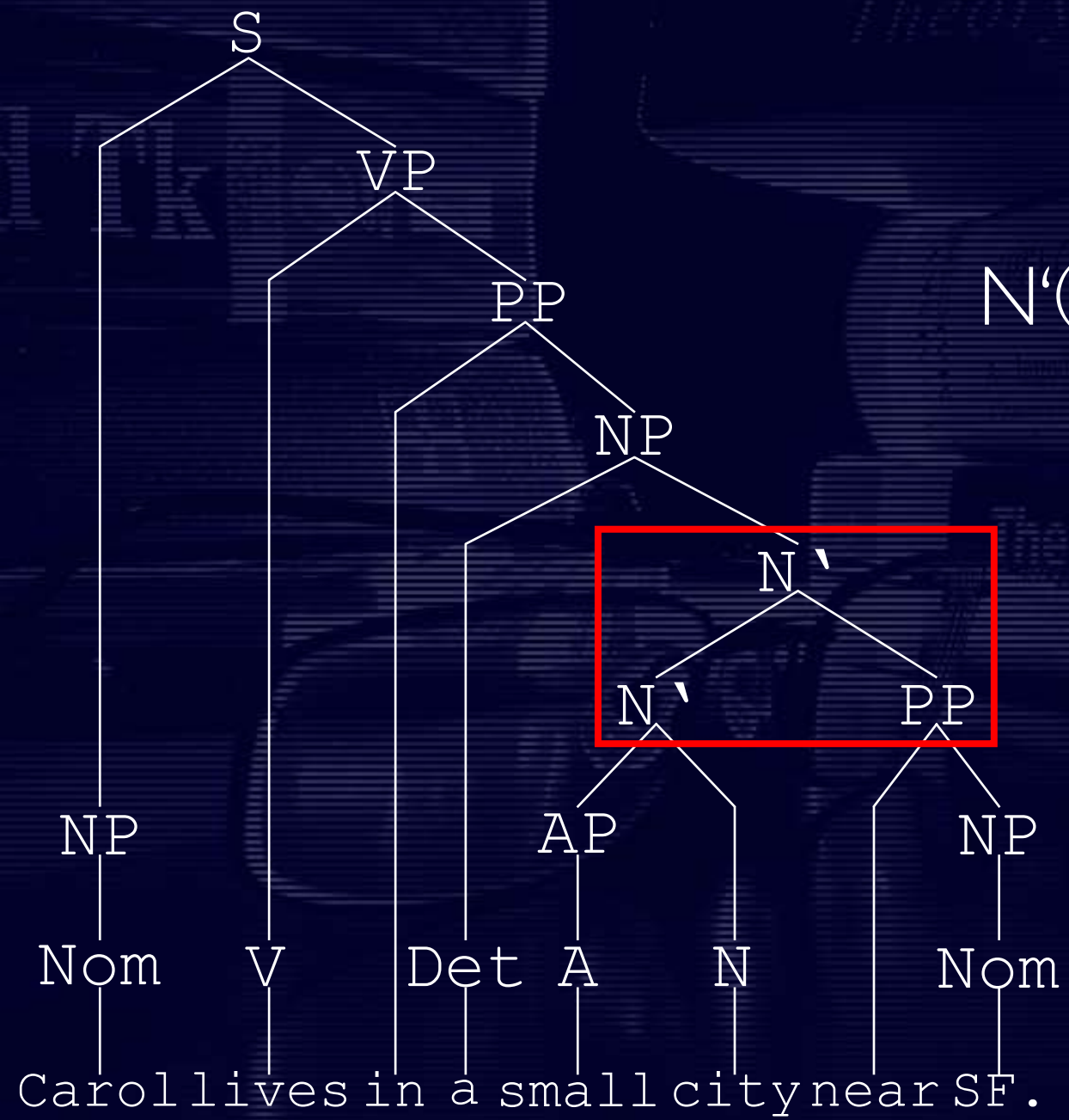
Compositionality



N' -> AP N

N'(**x**) := T(AP(**x**), N(**x**))

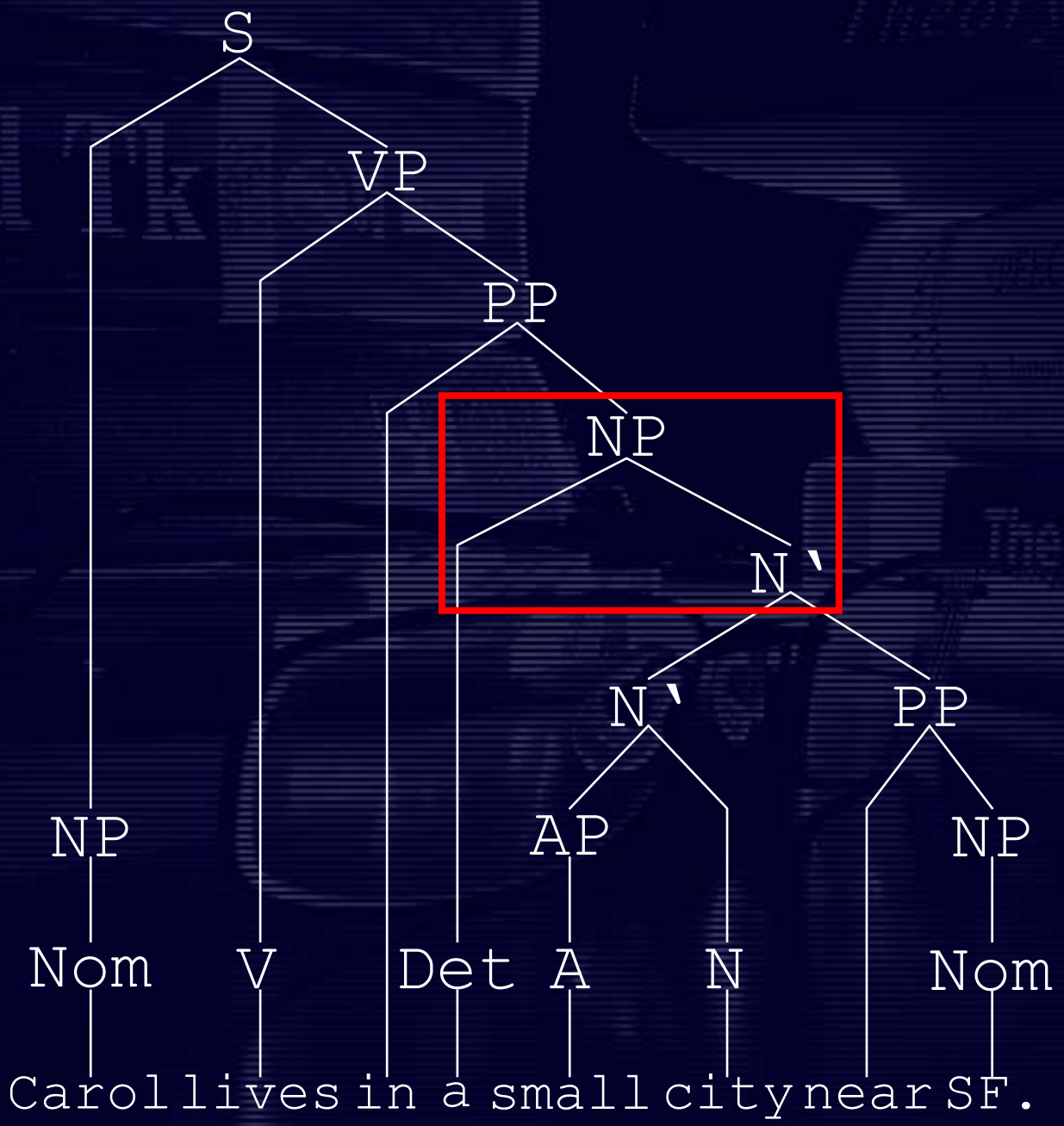
Compositionality



N' -> N' PP

N‘(**x**) := T(N‘(**x**), PP‘(**x**))

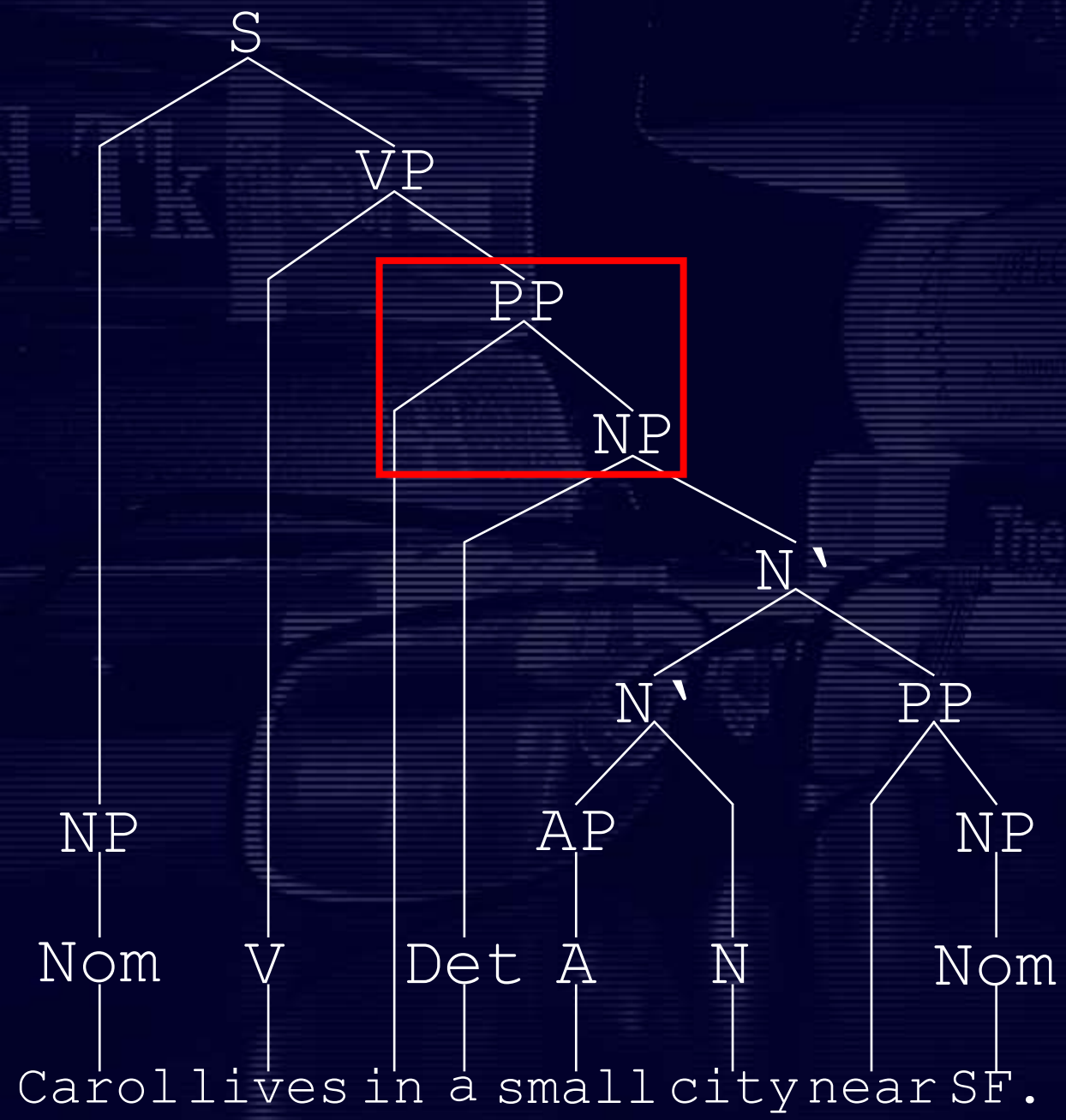
Compositionality



NP -> Det N'

NP(**x**) := N'(**x**)

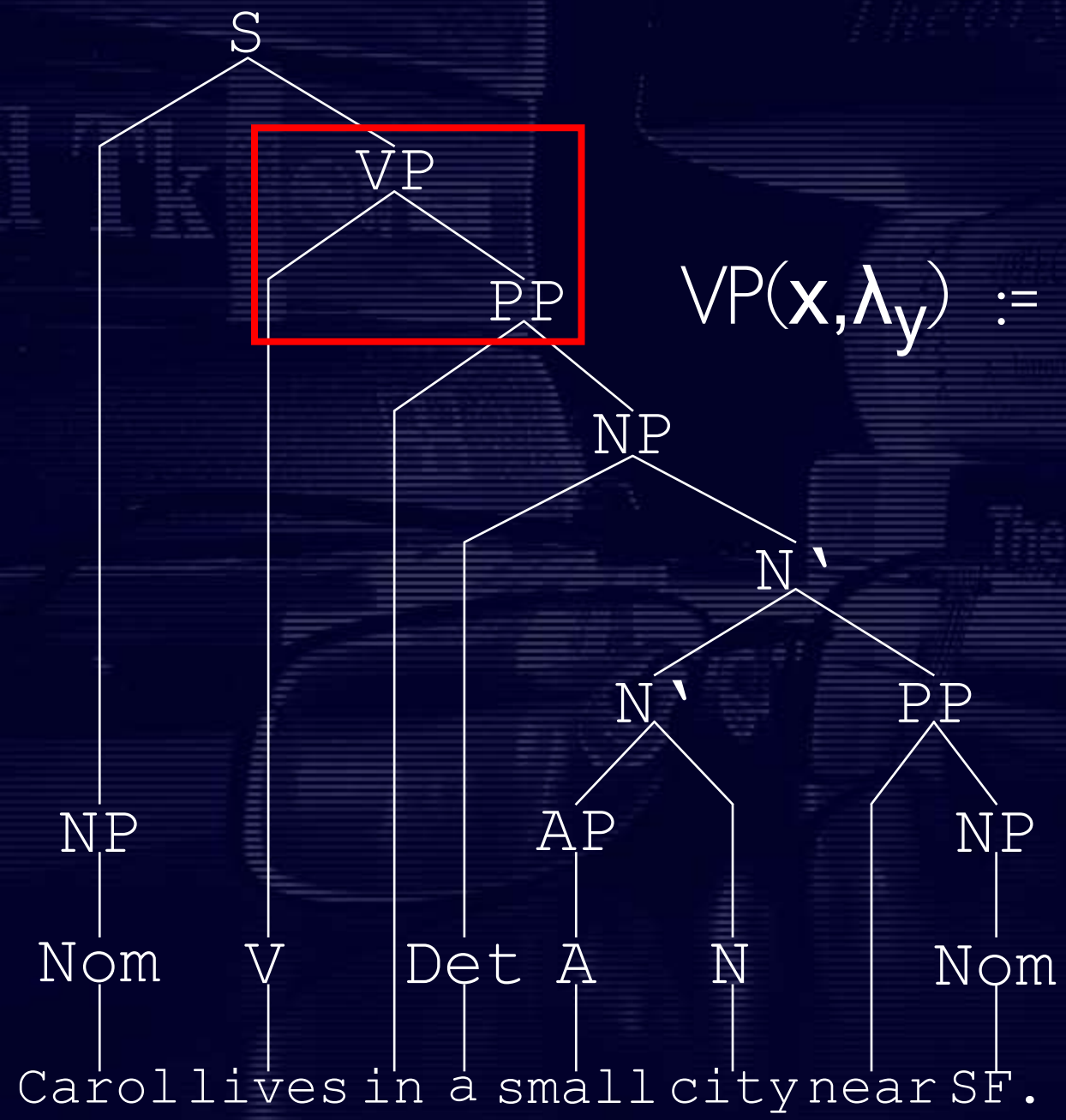
Compositionality



PP -> in NP

PP(**x**) := NP(**x**)

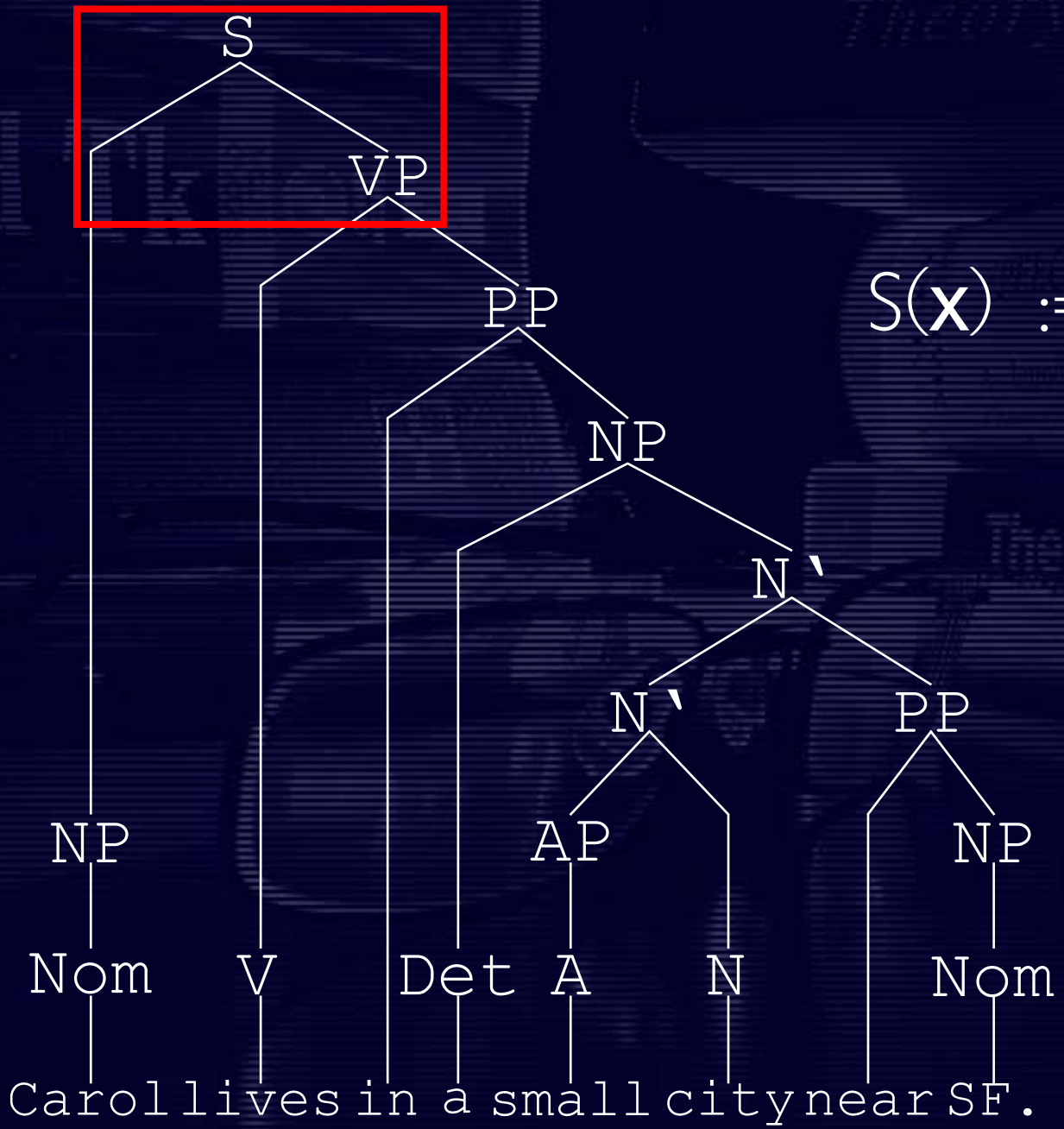
Compositionality



VP -> V PP

$VP(\mathbf{x},\lambda_y) := \sup_z\{T(V(\mathbf{x},\lambda_y,\mathbf{z}),PP(\mathbf{z}))\}$

Compositionality



S -> NP VP

$S(\mathbf{x}) := \sup_{\mathbf{y}}\{T(NP(\mathbf{y}),VP(\mathbf{x},\mathbf{y}))\}$

Concluding Remarks

Using our semantic grammar, we derived a fuzzy set S(**x**) of all records **x** in a data-base where ***Carol lives in a small city near San Francisco.***

The universal semantics of this phrase is a weak ordering of such records **x**, ranked according to their membership degrees.

As this is needed in a PNL, we believe our contributions to be a relevant step in Lotfi Zadeh's new direction of AI.

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